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ON THE GAUSS MAP OF MINIMAL SURFACES IMMERSED IN $\mathbf{R}^n$

MIN RU

Abstract

In this paper, we prove that the Gauss map of a nonflat complete minimal surface immersed in a Euclidean n-space $\mathbf{R}^n$ can omit at most $n(n+1)/2$ hyperplanes in a complex projective $(n-1)$-space $\mathbf{C}P^{n-1}$ located in general position.

1. Introduction

Let M be a smooth oriented two-manifold without boundary. Take an immersion $f : M \to \mathbf{R}^n$. The metric on M induced from the standard metric ds_E^2 on $\mathbf{R}^n$ by f is denoted by ds^2. Let Δ denote the Laplace-Beltrami operator of (M, ds^2). The local coordinates (x, y) on (M, ds^2) are called *isothermal* if $ds^2 = h(dx^2 + dy^2)$ for some local function $h > 0$. Make M into a Riemann surface by decreeing that the 1-form $dx + idy$ is of type $(1, 0)$, where (x, y) are any isothermal coordinates. In terms of the holomorphic coordinate $z = x + iy$, we can write

$$\Delta = \frac{-4}{h}\frac{\partial^2}{\partial z \partial \bar{z}}.$$

We say that f is *minimal* if $\Delta f = 0$, i.e., an immersion into $\mathbf{R}^n$ is minimal if and only if it is harmonic relative to the induced metric.

The Gauss map of f is defined to be

$$G : M \to \mathbf{C}P^{n-1}, \qquad G(z) = [(\partial f/\partial z)],$$

where $[(\cdot)]$ denotes the complex line in $\mathbf{C}^n$ through the origin and $(\cdot)$. By the assumption of minimality of M, G is a holomorphic map of M into $\mathbf{C}P^{n-1}$.

In 1981, F. Xavier showed that the Gauss map of a nonflat complete minimal surface in $\mathbf{R}^3$ cannot omit seven points of the sphere [15]. In 1988, Fujimoto reduced seven to five, which is sharp [6]. For the $n > 3$

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