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HYPERBOLIZATION OF POLYHEDRA

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Introduction

Hyperbolization is a process for converting a simplicial complex into a metric space with "nonpositive curvature" in the sense of Gromov. Several such processes are described in [19, §3.4]. One of the purposes of this paper is to elaborate this idea of Gromov. Another purpose is to use it to construct the three examples described below.

Our approach to hyperbolization is based on the following construction of Williams [32]. Suppose that X is a space and that $f\colon X \to \sigma^n$ is a map onto the standard n-simplex. Suppose, also, that K is an n-dimensional simplicial complex. To these data Williams associates a space $X\Delta K$, constructed by replacing each n-simplex in the barycentric subdivision of K by a copy of X. The pair (X, f) is a "hyperbolized n-simplex" if X^n is a nonpositively curved manifold with boundary and f has appropriate properties. (It is proved in §4 that hyperbolized simplices exist.) If (X, f) is a hyperbolized n-simplex, then $X\Delta K$ is nonpositively curved; it is called a "hyperbolization of K."

In all three examples we begin with a polyhedral homology manifold having a desired feature; a hyperbolization then has the added feature of nonpositive curvature. The first example is a closed aspherical four-manifold which cannot be triangulated. Taking the product of this example with a n-torus, we obtain an aspherical manifold of any dimension ≥ 4 which is not homotopy equivalent to a PL manifold. The second example is a closed smooth manifold of dimension $n \geq 5$ which carries a topological metric of nonpositive curvature, while its universal cover, though contractible, is not homeomorphic to a Euclidean n-space $\mathbb{R}^n$. As we shall see, such a manifold cannot carry a PL or smooth metric of nonpositive curvature. The third example is a further refinement: M^n carries a topological metric of strict negative curvature and its universal

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