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ON THE GLOBAL EXISTENCE AND THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO THE EINSTEIN-MAXWELL-YANG-MILLS EQUATIONS

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Abstract

The Einstein equations coupled to massless (conformally invariant) source field equations are represented by conformal field equations which are regular for any sign of the conformal factor. For the case of four-dimensional Lorentz spaces we introduce an improved technique to reduce Cauchy problems for the conformal Einstein-Maxwell-Yang-Mills equations to Cauchy problems for symmetric hyperbolic systems. In the case where the sign of the cosmological constant is such that conformal infinity is space-like a general stability result for asymptotically simple solutions, a semiglobal existence result for asymptotically simple solutions obtained from arbitrary asymptotic initial data, and a global existence result for asymptotically simple solutions to the Einstein-Maxwell-Yang-Mills equations for data close to De-Sitter data is derived. In the case of vanishing cosmological constant it is shown that hyperboloidal initial sufficiently close to Minkowskian hyperboloidal initial data evolve into a solution of the Einstein-Maxwell-Yang-Mills equations which has a smooth asymptotic structure in future null directions and which is regular at future time-like infinity.

1. Introduction

In this article we generalize results obtained in [13], [16], [17] on the global, respectively semiglobal, existence and the asymptotic behavior of solutions to Einstein's field equations in the case of dimension four and Lorentz signature. We consider more general situations than before, present an improved technique to derive the results which exhibits also more clearly those properties of Einstein's field equations which determine the asymptotic behavior of the solutions, and include "matter fields" into the discussion. The possibility of some of these generalizations has been pointed out already in [17].

Our results are based on an analysis of the "conformal structure" of Einstein's field equations, i.e., on a study of the "conformal Einstein

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