

HOMOTOPY K3 SURFACES CONTAINING $\Sigma(2, 3, 7)$

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1. Introduction

An interesting question in 4-dimensional topology is whether each irreducible simply connected smooth 4-manifold other than S^4 must admit a complex structure. One technique which has been suggested for answering this question is to try to produce examples which have all of their Donaldson polynomials [3] vanishing, and then use Donaldson's theorem that complex algebraic surfaces have nontrivial polynomial invariants. A natural place to begin study is among smooth manifolds with the homotopy type of a K3 surface; we refer to such manifolds as homotopy K3 surfaces. Kodaira [7] has produced a family of homotopy K3 surfaces by performing logarithmic transforms on the fibers of elliptic K3's but these manifolds all have complex structures. (Their diffeomorphism types have been studied recently by Friedman and Morgan.) Also, there were many (unpublished) examples of homotopy K3 surfaces constructed about a decade ago by Kirby calculus pictures.

A common aspect of many of these latter examples is that they admit an embedding of the Brieskorn homology 3-sphere $\Sigma(2, 3, 7)$, which may be described as the link of a complex algebraic singularity $\{(x, y, z) \in \mathbb{C}^3 : x^2 + y^3 + z^7 = 0\} \cap S^5$, or, equivalently, as the result of -1 surgery on the right-handed trefoil knot. (The Poincaré homology sphere, $\Sigma(2, 3, 5)$, is the result of $+1$ surgery on the right-handed trefoil.) In this article we shall show

Theorem 1.1. *Any homotopy K3 surface which admits an embedding of $\Sigma(2, 3, 7)$ has a nontrivial Donaldson polynomial invariant of degree 10.*

We will give a fairly elementary proof of this fact based on Donaldson's study of 4-manifolds whose intersection form has one or two positive parts [2] and on our study of the representation space of $\Sigma(2, 3, 7)$ [4]. In particular, our calculations of Donaldson's invariant use no algebraic geometry.

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