J. DIFFERENTIAL GEOMETRY
34 (1991) 223-253

EXPLICIT SELF-DUAL METRICS ON $\mathbb{CP}_2 \# \cdots \# \mathbb{CP}_2$

CLAUDE LEBRUN

Abstract

We display explicit half-conformally-flat metrics on the connected sum of any number of copies of the complex projective plane. These metrics are obtained from magnetic monopoles in hyperbolic 3-space by an analogue of the Gibbons-Hawking ansatz, and are conformal compactifications of asymptotically-flat, scalar-flat Kähler metrics on n-fold blow-ups of $\mathbb{C}^2$. The corresponding twistor spaces are also displayed explicitly, and are observed to be Moishezon manifolds— that is, they are bimeromorphic to projective varieties.

1. Introduction

Motivated by examples due to Poon [25], Donaldson and Friedman [7] have proved the existence of self-dual conformal metrics on the connected sum

$$n\mathbb{CP}_2 := \underbrace{\mathbb{CP}_2 \# \cdots \# \mathbb{CP}_2}_{n}$$

of any number of copies of the complex projective plane. (Here a Riemannian metric on an oriented 4-manifold is called *self-dual* if its Weyl curvature, considered as a bundle-valued 2-form, is in the $+1$ eigenspace of the Hodge star operator; an orientable Riemannian 4-manifold is called *half-conformally-flat* if this holds for at least one orientation.) Their method involves a delicate desingularization of a singular model of the desired twistor space. An analytic argument for this existence theorem has also been given by Floer [8].

In this paper, we will obtain stronger results by more elementary methods. In fact, we will write down such metrics explicitly for each value of n by looking only for metrics with an S^1-symmetry, and observe that, in contrast to their generic deformations, the twistor spaces of the constructed metrics are *Moishezon*, meaning that they are bimeromorphically equivalent to projective-algebraic varieties, and are thus themselves abstract-

Received October 30, 1989 and, in revised form, May 10, 1990. The author's research was supported in part by National Science Foundation Grant DMS-8704401.