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A GEOMETRIC CHARACTERIZATION OF NEGATIVELY CURVED LOCALLY SYMMETRIC SPACES

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Introduction

Let M be a compact connected Riemannian manifold of nonpositive sectional curvature. In [1]-[3] the rank of M is defined as follows: For $v \in T^1M$ let the rank of v be the dimension of the vector space of parallel Jacobi fields along the geodesic γ_v with initial velocity v, and let $\mathrm{rank}(M)$ be the infimum of the rank of the elements of T^1M. Ballmann, Brin, Eberlein, and Spatzier showed [1]-[3] (compare also [5], [6]) that if M is irreducible (i.e., the de Rham decomposition of the universal covering of M is trivial), and $\mathrm{rank}(M) \geq 2$, then M is locally symmetric of higher rank.

As the rank of M measures its flatness, we can define a notion of rank for general manifolds of nonpositive curvature which measures the distribution of the curvature maximum in the following way: Let $-a^2 \leq 0$ be the maximum of the curvature of M. For an element v of the unit tangent bundle T^1M of M define the *hyperbolic rank* of v to be the dimension of the vector space of parallel vector fields J along the geodesic γ_v with initial velocity $\gamma_v'(0) = v$ with the following properties:

(1) J is orthogonal to the tangent of γ_v.

(2) For every $t \in \mathbf{R}$ the curvature of the plane spanned by $\gamma_v'(t)$ and $J(t)$ equals $-a^2$.

Let the *hyperbolic rank* $h\text{-}\mathrm{rank}(M)$ of M be the minimum of the hyperbolic ranks of the vectors $v \in T^1M$. It is not difficult to see that $\mathrm{rank}(M) = h\text{-}\mathrm{rank}(M) + 1$ for manifolds with curvature maximum 0.

With this notion of rank the result of Ballmann, Brin, Eberlein, and Spatzier holds for every manifold of nonpositive curvature:

Theorem. *If $h\text{-}\mathrm{rank}(M) > 0$ and M is irreducible, then M is locally symmetric.*

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