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THE SMALLEST HYPERBOLIC 3-MANIFOLDS WITH TOTALLY GEODESIC BOUNDARY

SADAYOSHI KOJIMA & YOSUKE MIYAMOTO

Dedicated to Professor Akio Hattori on his sixtieth birthday

0. Introduction

A hyperbolic manifold is a riemannian manifold with constant sectional curvature -1. Jørgensen showed that there are only finitely many topological types of the thick part of complete hyperbolic 3-manifolds with bounded volume. Thurston then showed that almost every Dehn surgery on a cusped manifold yields a hyperbolic manifold, and their volumes accumulate at the original cusped manifold from below. These results lead to a global description of the volumes of hyperbolic 3-manifolds which form a well-ordered set of order type ω^ω in several situations [11].

Particular interest has been taken by various authors in the minimum volume. Among others, Meyerhoff [9], Adams [1], and Chinburg and Friedman [3] found the cusped 3-orbifold, the cusped 3-manifold, and the arithmetic 3-orbifold of minimal volume, respectively. In this paper, we will prove

Theorem. *Among compact hyperbolic 3-manifolds with nonempty totally geodesic boundary, each one having the minimum volume admits a polyhedral decomposition by two regular truncated tetrahedra of dihedral angle $\pi/6$.*

The minimum is hence twice the volume of a regular truncated tetrahedron of dihedral angle $\pi/6$. It can be expressed by a definite integral of some elementary functions, and the numerical computation shows that it is $6.452\cdots$. The reader is asked to compare this large value with the other minima. A manifold having the minimum volume is necessarily orientable but not unique, and those manifolds are described by Thurston in [11] and classified by Fujii [5].

We review the polyhedral decomposition in the next section. In §2, we describe the minimum volume, the manifold shown to have the minimum, and its rigidity property in terms of the shape of cut locus. In §3,

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