J. DIFFERENTIAL GEOMETRY
34 (1991) 169-174

ESTIMATE OF THE SINGULAR SET OF THE EVOLUTION PROBLEM FOR HARMONIC MAPS

XIAOXI CHENG

1. Introduction

Let $\mathscr{M}$, $\mathscr{N}$ be Riemannian manifolds of dimensions m, n $(m > 2)$ with metrics γ, g respectively. We consider the evolution of harmonic maps [3, (1.4)], [1,(1.6), (1.7)]:

$$\partial_t u - \Delta_{\mathscr{M}} u + \Gamma_{\mathscr{N}}(u)(\nabla u, \nabla u)_{\mathscr{M}} = 0, \qquad u|_{t=0} = u_0. \tag{1.1}$$

M. Struwe proved the following theorem.

[3, Theorem 6.1]. *Suppose $u: \mathbf{R}^m \times \mathbf{R}_+ \to \mathscr{N}$ is the limit of a sequence u_k of regular solutions to* (1.1), *with finite energy*

$$E(u_k(t)) \le E_0 < \infty, \quad \forall k \in \mathbf{N} \text{ and } t > 0$$

in the sense that $E(u(t)) \le E_0$ almost everywhere and that $\nabla u_k \to \nabla u$ weakly in $L^2(Q)$ for any compact $Q \subset \mathbf{R}^m \times \mathbf{R}_+$. Then u solves (1.1) *in the classical sense and is regular on a dense open subset of $\mathbf{R}^m \times \mathbf{R}_+$ whose complement Σ has locally finite m-dimensional Hausdorff measure* (*with respect to the parabolic metric*).

Here we give a better estimate on the singular set Σ.

Theorem. *If $t_0 > 0$, then $\Sigma \cap (\mathbf{R}^m \times \{t_0\})$ has finite $(m-2)$-dimensional Hausdorff measure.*

Remarks. In [1], with a general m-dimensional Riemannian manifold $\mathscr{M}$ replacing $\mathbf{R}^m$, Y. Chen and M. Struwe proved the *existence* of a solution to (1.1), which satisfies all the above conditions of [3, Theorem 6.1]. Here E_0 is the energy of the initial map $u(\cdot, 0)$.

In the case $m = 2$, M. Struwe [2] proved that Σ consists of at most finitely many points of $\mathscr{M} \times \mathbf{R}_+$.

Received March 2, 1990.