J. DIFFERENTIAL GEOMETRY
34 (1991) 165-168

SOME NEW HARMONIC MAPS FROM B^3 TO S^2

CHI-CHEUNG POON

I. Introduction

It is well known that $u_0(x) = x/|x|$ is the unique minimizer of the energy functional $\int_{B^3} |\nabla u|^2\, dx$ among maps $u \in H^1(B^3, S^2)$ such that $u(x) = x$ for $x \in \partial B^3$ [2] where B^3 and S^2 are the unit 3-ball and 2-sphere respectively. Such an energy minimizer is a *weakly harmonic* map [4]. By minimizing a "relaxed energy", F. Bethuel, H. Brezis, and J.-M. Coron [1] proved that there exist infinitely many weakly harmonic maps for any nonconstant boundary data. But the regularity of such weakly harmonic maps is still unknown. Here we use a different approach to obtain the following result.

Theorem. *For any x_0 in $\overline{B}^3$, there is a harmonic map $u\colon B^3 \to S^2$ such that*

(i) *$u(x) = x$ on ∂B^3;*

(ii) *u is smooth in $\overline{B}^3 \sim \{x_0\}$, i.e., x_0 is the only singularity of u.*

Let r, α, and z be cylindrical coordinates in $\mathbf{R}^3$, i.e., $x = r\cos\alpha$, $y = r\sin\alpha$. A map $u\colon B^3 \to S^2$ is called, as in [5], *axially symmetric* if in r, α, z

$$u(r, \alpha, z) = (\cos\alpha \sin\varphi, \sin\alpha\sin\varphi, \cos\varphi) \tag{1}$$

for some real valued function $\varphi(r, z)$. Using (1), we can simplify the formula for the energy of an axially symmetric map u,

$$\int_{B^3} |\nabla u|^2\, dx = 2\pi \int_D r\left(\frac{\partial\varphi}{\partial r}\right) + r\left(\frac{\partial\varphi}{\partial z}\right)^2 + \frac{\sin^2\varphi}{r}\, dr\, dz,$$

where $D = \{(r, z) : r^2 + z^2 < 1,\ r > 0\}$.

For any smooth $\varphi\colon D \to \mathbf{R}$, define

$$E(\varphi) = 2\pi \int_D r\left(\frac{\partial\varphi}{\partial r}\right)^2 + r\left(\frac{\partial\varphi}{\partial z}\right)^2 + \frac{\sin^2\varphi}{r}\, dr\, dz.$$

Received December 18, 1989 and, in revised form, February 22, 1990.