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# THE LOCAL BEHAVIOUR OF HOLOMORPHIC CURVES IN ALMOST COMPLEX 4-MANIFOLDS

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**Abstract**

In this paper we prove various results about the positivity of intersections of holomorphic curves in almost complex 4-manifolds which were stated by Gromov. We also show that the virtual genus of any closed holomorphic curve in an almost complex 4-manifold is nonnegative. These technical results form the basis of the classification of rational and ruled symplectic 4-manifolds given in [5].

## 1. Introduction

Let $(V, J)$ be an almost complex manifold, and let $(\Sigma, J_0)$ be a Riemann surface. A (parametrized) *J-holomorphic curve* in $(V, J)$ is a map $f: \Sigma \to V$ which preserves the almost complex structures, i.e., $df \circ J_0 = J \circ df$. The unparametrized curve $\operatorname{Im} f$ is denoted by $C$. We will consider both local and global questions. In the former case, $C$ will be the image of a disc $D$ centered at the origin in the complex plane $\mathbb{C}$, and in the latter it will be closed, i.e., the image of a compact Riemann surface $\Sigma$ without boundary. Throughout, we will assume that $(V, J)$ and all maps are $C^\infty$-smooth, unless there is explicit mention to the contrary. Further, we will assume that $f$ is not a multiple covering, i.e., that $f$ does not factor as $f' \circ \gamma$, where $\gamma: \Sigma \to \Sigma$ is a $J_0$-holomorphic self-map of degree $> 1$.

Our first aim is to prove the following result, which was stated by Gromov in [3, 2.1.$C_2$].

**Theorem 1.1.** *Two closed distinct $J$-holomorphic curves $C$ and $C'$ in an almost complex 4-manifold $(V, J)$ have only a finite number of intersection points. Each such point $x$ contributes a number $k_x \geq 1$ to the algebraic intersection number $C \cdot C'$. Moreover, $k_x = 1$ only if the curves $C$ and $C'$ intersect transversally at $x$.*

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