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NODAL GEOMETRY ON RIEMANNIAN MANIFOLDS

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1. Let M^n be a smooth, compact, and connected Riemannian manifold with no boundary. Let Δ denote the Laplacian on M. Let $-\Delta u = \lambda u$, u an eigenfunction with eigenvalue λ, $\lambda > 1$. Our main theorems are:

Theorem 1 (BMO estimate for $\log|u|$). *For u, λ as above,*

$$\| \log |u| \|_{\mathrm{BMO}} \leq c\lambda^n \log \lambda,$$

where c is independent of λ, and depends only on n and M.

Theorem 2 (Geometry of nodal domains). *Let u, λ be as above, let $B \subset M$ be any ball, and let $\Omega \subset B$ be any of the connected components of $\{x \in B : u(x) \neq 0\}$. If Ω intersects the middle half of B, then*

$$|\Omega| \geq c\lambda^{-2n^2-n/2}(\log \lambda)^{-2n}|B|,$$

where c is independent of λ and u.

Similar theorems have been proved by H. Donnelly and C. Fefferman [1], [2] with $\lambda^n \log \lambda$ replaced by $\lambda^{n(n+2)/4}$ in Theorem 1 and $\lambda^{-2n^2-n/2}(\log \lambda)^{-2n}$ replaced by $\lambda^{-(n+n^2(n+2))/2}$ in Theorem 2. Of course, it is obvious that Theorems 1 and 2 above are not best possible.

Theorem 1 is the key to Theorem 2. We deduce Theorem 2 from Theorem 1 by essentially following the arguments in [2] with appropriate modifications in view of the better BMO estimate of Theorem 1.

We shall use the symbols c, c_0, c_1, c_2, c_3, c_4, and $\bar{c}$ to denote generic constants which are independent of λ.

2. Before commencing the proof of Theorem 1, we recall two facts from [2]. We state these as Theorem 0.

Theorem 0. *Let M, u, λ be as above. Let $B(x, \delta)$ denote the ball centered at x of radius δ. Then*

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