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SURFACES AND BRANCHED SURFACES TRANSVERSE TO PSEUDO-ANOSOV FLOWS ON 3-MANIFOLDS

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Abstract

Given a circular pseudo-Anosov flow φ on an irreducible, atoroidal 3-manifold M, we classify all closed surfaces in M which are transverse and "almost transverse" to φ, generalizing the Schwartzmann-Fried classification of cross-sections to φ. In particular, there exists an "almost transverse" surface representing any class in $H_2(M; \mathbf{Z})$ which is nonnegative on all homology directions of φ. As an application, if σ is a fibered face of the unit ball of Thurston's polyhedral norm on $H_2(M; \mathbf{R})$, we give conditions under which Oertel's conjecture can be verified, that there exists a single taut branched surface in M carrying norm-minimizing representatives of every class in $\operatorname{Cone}(\sigma)$, and in particular carrying fiber representatives of every class in $\operatorname{int}(\operatorname{Cone}(\sigma))$.

0. Introduction

The study of fibrations of 3-dimensional manifolds over the circle gained great impetus with the introduction in [12] of Thurston's norm on the homology and cohomology of a 3-manifold. The norm x on $H_2(M; \mathbf{R})$ is defined in the following manner. Given $\alpha \in H_2(M; \mathbf{Z}) \subset H_2(M; \mathbf{R})$, $x(\alpha)$ is defined as the infimum, over all embedded surfaces A representing α, of

$$\chi_-(A) = -\chi(A - \text{spherical components of } A).$$

x is then extended by homogeneity and continuity to all of $H_2(M; \mathbf{R})$. In general, x is only a seminorm, but if M has no nonseparating spheres or tori, and in particular when M is irreducible and atoroidal, then x is a norm. Thurston showed that the unit ball $B_x = B_x(M)$ of x is always a polyhedron with integrally defined faces. Moreover, there is a certain collection of top-dimensional faces of B_x, called the fibered faces, such that a class $\alpha \in H_2(M; \mathbf{Z}) \subset H_2(M; \mathbf{R})$ is represented by a fiber of some

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