SURFACES AND BRANCHED SURFACES
TRANSVERSE TO PSEUDO-ANOSOV FLOWS
ON 3-MANIFOLDS

LEE MOSHER

Abstract
Given a circular pseudo-Anosov flow \( \varphi \) on an irreducible, atoroidal 3-manifold \( M \), we classify all closed surfaces in \( M \) which are transverse and "almost transverse" to \( \varphi \), generalizing the Schwartzmann-Fried classification of cross-sections to \( \varphi \). In particular, there exists an "almost transverse" surface representing any class in \( H_2(M; \mathbb{Z}) \) which is nonnegative on all homology directions of \( \varphi \). As an application, if \( \sigma \) is a fibered face of the unit ball of Thurston's polyhedral norm on \( H_2(M; \mathbb{R}) \), we give conditions under which Oertel's conjecture can be verified, that there exists a single taut branched surface in \( M \) carrying norm-minimizing representatives of every class in \( \text{Cone}(\sigma) \), and in particular carrying fiber representatives of every class in \( \text{int}(\text{Cone}(\sigma)) \).

0. Introduction
The study of fibrations of 3-dimensional manifolds over the circle gained great impetus with the introduction in [12] of Thurston's norm on the homology and cohomology of a 3-manifold. The norm \( x \) on \( H_2(M; \mathbb{R}) \) is defined in the following manner. Given \( \alpha \in H_2(M; \mathbb{Z}) \subset H_2(M; \mathbb{R}) \), \( x(\alpha) \) is defined as the infimum, over all embedded surfaces \( A \) representing \( \alpha \), of

\[
\chi_-(A) = -\chi(A - \text{spherical components of } A).
\]

\( x \) is then extended by homogeneity and continuity to all of \( H_2(M; \mathbb{R}) \). In general, \( x \) is only a seminorm, but if \( M \) has no nonseparating spheres or tori, and in particular when \( M \) is irreducible and atoroidal, then \( x \) is a norm. Thurston showed that the unit ball \( B_x = B_x(M) \) of \( x \) is always a polyhedron with integrally defined faces. Moreover, there is a certain collection of top-dimensional faces of \( B_x \), called the fibered faces, such that a class \( \alpha \in H_2(M; \mathbb{Z}) \subset H_2(M; \mathbb{R}) \) is represented by a fiber of some

Received November 9, 1988, and, in revised form, May 17, 1990. The author was funded by a National Science Foundation postdoctoral research fellowship.