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GEOMETRIC QUANTIZATION OF CHERN-SIMONS GAUGE THEORY

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Abstract

We present a new construction of the quantum Hilbert space of Chern-Simons gauge theory using methods which are natural from the three-dimensional point of view. To show that the quantum Hilbert space associated to a Riemann surface Σ is independent of the choice of complex structure on Σ, we construct a natural projectively flat connection on the quantum Hilbert bundle over Teichmüller space. This connection has been previously constructed in the context of two-dimensional conformal field theory where it is interpreted as the stress energy tensor. Our construction thus gives a $(2+1)$-dimensional derivation of the basic properties of $(1+1)$-dimensional current algebra. To construct the connection we show generally that for affine symplectic quotients the natural projectively flat connection on the quantum Hilbert bundle may be expressed purely in terms of the intrinsic Kähler geometry of the quotient and the Quillen connection on a certain determinant line bundle. The proof of most of the properties of the connection we construct follows surprisingly simply from the index theorem identities for the curvature of the Quillen connection. As an example, we treat the case when Σ has genus one explicitly. We also make some preliminary comments concerning the Hilbert space structure.

Introduction

Several years ago, in examining the proof of a rather surprising result about von Neumann algebras, V. F. R. Jones [20] was led to the discovery of some unusual representations of the braid group from which invariants of links in S^3 can be constructed. The resulting "Jones polynomial" of links has proved in subsequent work to have quite a few generalizations, and to be related to two-dimensional lattice statistical mechanics and to quantum groups, among other things.

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