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UNIQUENESS AND EXISTENCE OF VISCOSITY SOLUTIONS OF GENERALIZED MEAN CURVATURE FLOW EQUATIONS

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1. Introduction

This paper treats degenerate parabolic equations of second order

$$u_t + F(\nabla u, \nabla^2 u) = 0 \tag{1.1}$$

related to differential geometry, where ∇ stands for spatial derivatives of $u = u(t, x)$ in $x \in \mathbb{R}^n$, and u_t represents the partial derivative of u in time t. We are especially interested in the case when (1.1) is regarded as an evolution equation for level surfaces of u. It turns out that (1.1) has such a property if F has a scaling invariance

$$F(\lambda p, \lambda X + \sigma p \otimes p) = \lambda F(p, X), \qquad \lambda > 0, \ \sigma \in \mathbb{R}, \tag{1.2}$$

for a nonzero $p \in \mathbb{R}^n$ and a real symmetric matrix X, where $\otimes$ denotes a tensor product of vectors in $\mathbb{R}^n$. We say (1.1) is *geometric* if F satisfies (1.2). A typical example is

$$u_t - |\nabla u| \operatorname{div}(\nabla u/|\nabla u|) = 0, \tag{1.3}$$

where ∇u is the (spatial) gradiant of u. Here $\nabla u/|\nabla u|$ is a unit normal to a level surface of u, so $\operatorname{div}(\nabla u/|\nabla u|)$ is its mean curvature unless ∇u vanishes on the surface. Since $u_t/|\nabla u|$ is a normal velocity of the level surface, (1.3) implies that a level surface of solution u of (1.3) moves by its mean curvature unless ∇u vanishes on the surface. We thus call (1.3) the *mean curvature flow equation* in this paper.

The motion of a closed (hyper)surface in $\mathbb{R}^n$ by its mean curvature has been studied by many authors [1], [3], [4], [8], [10], [12], [14], [15]. Such a motion is also important in the singular perturbation theory related to

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