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A SIMPLE CONSTRUCTION OF ATIYAH-SINGER CLASSES AND PIECEWISE LINEAR TRANSFORMATION GROUPS

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The main purpose of this paper is to provide a simple transparent construction of the Atiyah-Singer classes associated to semifree PL actions of finite groups with manifold fixed sets, such as the locally linear actions. These classes are so named because they enter into nonsmooth versions of the G-signature formula. We shall also see how they enter into existence and classification problems for group actions in a way that the smooth characteristic classes of [4] do not. Other papers which, in some cases implicitly, deal with these classes or some analogue of them (or their use in constructing actions) are, [6], [9], [10], [16], [18], [21], [26], [27], [30], [43]; they define them by a wide variety of techniques from homological surgery to sheaf theory to analytical methods more in the spirit of Atiyah and Singer. We hope that the present self-contained treatment in the simple contexts of semifree PL and of PL locally linear actions will help readers to understand the other treatments as well.

Actually there are some subtleties due to local linearity not present for more general PL actions. These are basically due to the fact that unrestricted coning is not permitted in this category. For instance, for $G = Z_{163}$ [40] (or $G = Z_n$ for n divisible by several primes [42]) the range of the G-signature map is different for PL locally linear actions than it is for general PL actions. Closer to the point of this paper, there is an additional obstruction for a submanifold of the sphere to be a semifree PL locally linear fixed set beyond those necessary for it to be a PL fixed set [41]. For this, the refinements obtained in this paper (not available in the cited papers) are crucial.

Thus approach is based on the analysis and comparison of classifying spaces $BSRN_k^G$ (respectively, $B\widetilde{SPL}_k^G$) for oriented equivariant PL

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