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RIGIDITY OF HOLOMORPHIC MAPS BETWEEN COMPACT HERMITIAN SYMMETRIC SPACES

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The purpose of this note is to prove the following.

Main Theorem. *Let X and Y be two equidimensional irreducible Hermitian symmetric spaces of compact type with* $\mathrm{rank}(Y) \geq 2$. *Then any holomorphic map f from X to Y is either a constant map or a biholomorphism.*

We briefly explain the motivation for studying this problem. Mok [3] studied uniqueness theorems of Hermitian metrics of seminegative curvature on quotients of bounded symmetric domain of rank ≥ 2. An immediate corollary of this theorem is the assertion that any nonconstant holomorphic map from M to N, where M denotes a compact quotient of bounded symmetric domain with rank ≥ 2 and N a Hermitian manifold of seminegative curvature, is in fact an isometric immersion. It is then natural to ask the analogous question in the case of compact type. However, due to the fact that in this case the automorphisms in general are not metric-preserving, we do not have the uniqueness of canonical metrics and therefore we can only try to prove that f is a holomorphic immersion instead of an isometric immersion. The correct formulation for the metric rigidity phenomenon in the case of compact type has been carried out by Mok [4]. As for the mapping rigidity, if we consider a holomorphic map of degree 2 from the quadric $\mathbf{Q}_n$ to $\mathbf{P}_n$ and an imbedding form $\mathbf{P}_n$ to $\mathbf{Q}_{2n}$, then the composite map fails to be an imbedding. To take into account this remark, we formulate our theorem in the equidimensional case. The cases of unequal dimensions remain open.

Our ideas for the proof can be sketched as follows. We look at a class of objects in a Hermitian symmetric space of compact type, i.e., the so-called characteristic spheres as in [4] or minimal rational curves as in [5]. The importance of the roles they play in the theory of Hermitian symmetric spaces has been illustrated before, as can be seen from the articles [4], [5].

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