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COMPACT CONSTANT MEAN CURVATURE SURFACES IN EUCLIDEAN THREE-SPACE

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1. Introduction

The main subject of this paper is the construction of closed CMC surfaces of any genus $g \geq 3$. The abbreviation "CMC surfaces" is used throughout the paper and stands for "properly immersed complete boundaryless surfaces in E^3 of constant mean curvature $H \equiv 1$". We also talk about "compact CMC surfaces", which means the same except "with boundary" rather than without. Compact CMC surfaces of any genus $g \geq 3$ with boundary a round planar circle are also constructed. These constructions are achieved by properly strengthening the methods employed in [10] to construct CMC surfaces with ends. The main results of this paper were announced in [8].

The question of whether such surfaces exist has a long history. In 1853 J. H. Jellett proved that star-shaped closed CMC surfaces are round spheres. In 1900 Liebmann [13] proved the same for convex surfaces. S.-S. Chern [3] extended Liebman's result to a certain class of convex W-surfaces. Hopf [5] established that any CMC topological sphere is round and asked whether the same is true for all closed CMC surfaces. Alexandrov [1] gave an affirmative answer for embedded surfaces. Wu-Yi Hsiang settled in the negative the higher dimensional analogue to Hopf's question [6]. Eventually, H. C. Wente [14] settled the so-called Hopf's conjecture also in the negative by constructing infinitely many CMC tori.

This paper is self-contained in the sense that the results presented here can be understood without reference to any other papers. However, many of the proofs are extensions of proofs in [10] and it would be impossible to make them self-contained without repeating most of that paper. Familiarity with [10] would be helpful also in understanding the basic idea of the construction which we proceed to outline.

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