ON THE ENTROPY ESTIMATE FOR THE RICCI FLOW ON COMPACT 2-ORBIFOLDS

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1. Introduction

In [3], Richard Hamilton began the study of the following equation, which we refer to as Hamilton’s Ricci flow:

\( \frac{\partial g}{\partial t}(x, t) = (r - R(x, t))g(x, t), \quad x \in M, \quad t > 0. \)

Here \( g \) is the metric, \( R \) is the scalar curvature of \( g \) (=twice the Gaussian curvature \( K \)), and \( r \) is the average of \( R \). Based on the work of Hamilton [3], and the subsequent extensions by the author [1] and Lang-Fang Wu [5], Wu and the author [2] recently proved the following, which was conjectured by Hamilton.

**Theorem 1.1.** If \((M, g)\) is a compact 2-dimensional Riemannian orbifold, then under Hamilton’s Ricci flow, \( g \) approaches asymptotically a Ricci soliton.

We say that \( \{g_t\} \) is a Ricci soliton if there exist diffeomorphisms \( \{\varphi_t\} \) of \( M \) such that \( g_t = \varphi_t(g_0) \).

The purpose of this note is to give a simple derivation of the entropy estimates which were used in the papers quoted above. This proof is based on an identity, and unlike the previous proofs, it does not use the fact that the solution to Hamilton’s Ricci flow exists for all time.

2. The entropy estimates

Let \((M, g)\) be a compact Riemannian 2-orbifold with positive Euler characteristic. In [3, §7], Hamilton showed that, provided the scalar curvature \( R \) of \( g \) is positive, the entropy \(-N(t)\) is increasing under \((\ast)\), where

\[ N(t) = \int_{M_t} R \log R \, dA. \]

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