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INFINITE ENERGY HARMONIC MAPS AND DEGENERATION OF HYPERBOLIC SURFACES IN MODULI SPACE

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1. Introduction

Let $(M, \sigma|dz|^2)$ be a closed, compact surface of genus g equipped with the (hyperbolic) metric σ of constant curvature $K \equiv -1$. If $\mathscr{M}_{-1}$ denotes the space of all constant curvature metrics on M, then both the group of orientation preserving diffeomorphisms Diff^+ and the group of diffeomorphisms isotopic to the identity Diff_0 act by pull-back on $\mathscr{M}_{-1}$; we define the Teichmüller space of genus g, T_g, to be the quotient space $\mathscr{M}_{-1}/\mathrm{Diff}_0$ and the moduli space of genus g, $\mathscr{M}_g$, to be the quotient space $\mathscr{M}_{-1}/\mathrm{Diff}^+$. Then $(M, \sigma|dz|^2)$ represents a point in the Teichmüller space T_g of surfaces of genus g. In [17], Sampson described a parametrization for a neighborhood of $(M, \sigma|dz|^2)$ in T_g, in terms of a neighborhood of zero in the vector space $QD(\sigma)$ of holomorphic quadratic differentials on $(M, \sigma|dz|^2)$. In [22], we used harmonic maps to derive an explicit asymptotic series for the hyperbolic metrics near $(M, \sigma|dz|^2)$ and, in Theorem 2.2, we shall show that this series converges, thus giving an explicit description of the real-analytic structure of the moduli space near an interior point $(M, \sigma|dz|^2)$.

The moduli space $\mathscr{M}_g$ admits a compactification $\overline{\mathscr{M}}_g$, which is a V-manifold without boundary; we use the notation $\mathscr{D}_g$ for the compactification divisor $\mathscr{D}_g = \overline{\mathscr{M}}_g \sim \mathscr{M}_g$. An element of $\mathscr{D}_g$ can be thought of as a Riemann surface with nodes, a connected complex space where points have neighborhoods complex isomorphic to either $\{|z| < \varepsilon\}$ (regular points) or $\{zw = 0; |z|, |w| < \varepsilon\}$ (nodes) and for which each component of the complement of the nodes has negative Euler characteristic.

Thus each component of the complement of the nodes admits a complete hyperbolic metric, with a deleted neighborhood of a node being isometric to two copies of a neighborhood of a hyperbolic cusp, given for

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