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EXISTENCE OF SMOOTH EMBEDDED SURFACES OF PRESCRIBED GENUS THAT MINIMIZE PARAMETRIC EVEN ELLIPTIC FUNCTIONALS ON 3-MANIFOLDS

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Introduction

Let F be a smooth positive function on the boundary of the unit ball in a Euclidean 3-space R^3. Then F defines a functional on immersed surfaces M in R^3 by the formula

$$F(M) = \int_M F(\nu)\, dA,$$

where $\nu(x)$ is the unit normal to M at x and the integration is with respect to surface area. Such a functional is called a "constant coefficient parametric functional". In this paper we study the existence of smooth embedded surfaces (with given boundaries) that minimize F. In particular, we show:

Theorem 3.4 (condensed version). *If F is even and elliptic, and S is a smooth simple closed curve on the boundary of a convex set in R^3, then for each $g \geq 0$ there exists a smooth embedded surface that minimizes $F(M)$ among all embedded surfaces M with boundary $\partial M = S$ and* genus$(M) \leq g$.

Here "F is even" means that $F(\nu) \equiv F(-\nu)$, i.e., that $F(M)$ does not depend on the orientation of M. Ellipticity of F means that the set

$$\{x\colon |x| F(x/|x|) \leq 1\}$$

is uniformly convex; this is equivalent to ellipticity of the Euler-Lagrange equations for the corresponding functional on graphs.

More generally, F can depend on position as well as unit normal direction:

$$F\colon R^3 \times \partial B^3 \to R^+,$$

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