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QUATERNIONIC KÄHLER 8-MANIFOLDS WITH POSITIVE SCALAR CURVATURE

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1. Introduction

Consider the subgroup of $SO(4n)$ consisting of unit quaternions acting on $\mathbb{H}^n \cong \mathbb{R}^{4n}$ by right multiplication. We denote the normalizer of this subgroup by $Sp(n)Sp(1)$, which is a maximal subgroup of $SO(4n)$ for $n \geq 2$. A quaternionic Kähler manifold M is a manifold of dimension $4n$, $n \geq 2$, with a Riemannian metric whose linear holonomy group is contained in $Sp(n)Sp(1)$. It is well known that any quaternionic Kähler manifold is Einstein, so there is a trichotomy according as the constant scalar curvature t of M is positive, negative, or zero. In the latter case M is hyper-Kähler in the sense that it is rendered Kähler by a family of complex structures parameterized by the 2-sphere S^2. However in all three cases there exists a bundle over M with fiber S^2 whose total space Z is a complex manifold. For $t > 0$, Z has a canonical Kähler structure, though in general M will not itself be a complex Kähler manifold.

Wolf [25] showed that each compact simple centerless Lie group G is the isometry group of a quaternionic Kähler symmetric space, equal to the conjugacy class of a three-dimensional subgroup of G determined by a highest root of its Lie algebra. These "Wolf spaces" constitute the only known complete examples for $t > 0$, and the present work is devoted to a proof that there are no others when $n = 2$:

Theorem 1.1. *A complete connected quaternionic Kähler eight-manifold with $t > 0$ is isometric to the quaternionic projective plane $\mathbb{HP}^2$, the complex Grassmannian $\mathbb{G}r_2(\mathbb{C}^4)$, or the exceptional space $G_2/SO(4)$.*

In §2 we summarize known facts regarding a quaternionic Kähler eight-manifold M with positive scalar curvature, and the complex Kähler manifold Z. The latter is known as the twistor space of M, because of similarities with four dimensions. Indeed, our approach parallels Hitchin's classification [10] of Kähler twistor spaces of self-dual Riemannian

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