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AN EXOTIC 4-MANIFOLD

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In [1] we have constructed a fake smooth structure on a contractible 4-manifold W^4 relative to boundary. This is a smooth manifold V with $\partial V = \partial W$ such that the identity map $\partial V \to \partial W$ extends to a homeomorphism but not to a diffeomorphism $V \to W$. This is a relative result in the sense that V itself is diffeomorphic to W, even though no such diffeomorphism can extend the identity map on the boundary. Here we strengthen this result by dropping the boundary hypothesis at the expense of slightly enlarging W: We construct two compact smooth 4-manifolds Q_1, Q_2 which are homeomorphic but not diffeomorphic to each other. In particular *no* diffeomorphism $\partial Q_1 \to \partial Q_2$ can extend to a diffeomorphism $Q_1 \to Q_2$.

Let Q_i^4, $i = 1, 2$, be the 4-manifolds obtained by attaching 2-handles to B^4 along knots K_i, $i = 1, 2$, with $+1$-framing (see Figures 1 and 2). Clearly Q_1 and Q_2 are homotopy equivalent to $\mathbf{C}P_0^2 = \mathbf{C}P^2 - \mathrm{int}(B^4)$, and it will be shown that $\partial Q_1 = \partial Q_2$.

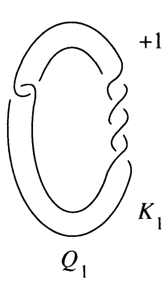

FIGURE 1

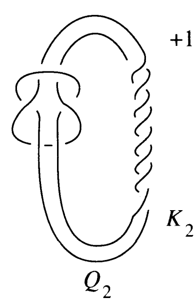

FIGURE 2

Theorem 1. *Q_1 and Q_2 are homeomorphic but not diffeomorphic to each other. In fact, even their interiors are not diffeomorphic to each other.*

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