J. DIFFERENTIAL GEOMETRY
33 (1991) 335-356

A FAKE COMPACT CONTRACTIBLE 4-MANIFOLD

SELMAN AKBULUT

Here we construct a fake smooth structure on a compact contractible 4-manifold W^4, where W^4 is a well-known Mazur manifold obtained by attaching in two-handle to $S^1 \times B^3$ along its boundary as in Figure 1.* Here we use the conventions of [2].

The results of this paper imply:

Theorem 1. *There is a smooth contractible 4-manifold V with $\partial V = \partial W$, such that V is homeomorphic but not diffeomorphic to W relative to the boundary.*

Let α be the loop in ∂W given by $S^1 \times p \subset S^1 \times S^2 = \partial(S^1 \times B^3)$ as in Figure 2. Zeeman raised the question whether α is slice in W [12], i.e., if α bounds an imbedded smooth D^2 in W. Even though it turned out that α is slice in another smooth contractible manifold with the same boundary [2], the original question has remained open. Let $f\colon \partial W \to \partial W$ be the diffeomorphism, obtained by first surgering $S^1 \times B^3$ to $B^2 \times S^2$ in the interior of W, then surgering the other imbedded $B^2 \times S^2$ back to $S^1 \times B^3$ (i.e., replacing the dots in Figure 2.)

Clearly this diffeomorphism extends to a self-homotopy equivalence of W. In fact, by [9], f extends to a homeomorphism $F\colon W \to W$. In [2, p. 279] the question of whether f extends to a diffeomorphism of W was posed. If it did, α would be slice in W since $f(\alpha)$ is clearly slice in W. Here we answer these questions negatively:

Theorem 2. *α is not slice in W, in particular f does not extend to a self-diffeomorphism of W.*

Theorem 1 follows from Theorem 2 as follows: Let $F\colon W \to W$ be a homeomorphism extending f. Let V be the smooth structure on W obtained by pulling back the smooth structure of W by F. This gives a diffeomorphism $F\colon V \to W$ extending f on the boundary. If $G\colon W \to V$

Received February 12, 1989 and, in revised form, July 10, 1989. The author was supported in part by National Science Foundation funds.

* Numbered figures appear at the end of the article (pp. 345-355).