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THE RICCI FLOW ON THE 2-SPHERE

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1. Introduction

The classical uniformization theorem, interpreted differential geometrically, states that any Riemannian metric on a 2-dimensional surface is pointwise conformal to a constant curvature metric. Thus one can consider the question of whether there is a natural evolution equation which conformally deforms any metric on a surface to a constant curvature metric. The primary interest in this question is not so much to give a new proof of the uniformization theorem, but rather to understand nonlinear parabolic equations better, especially those arising in differential geometry. A sufficiently deep understanding of parabolic equations should yield important new results in Riemannian geometry.

The question in the preceding paragraph has been studied by Richard Hamilton [3] and Brad Osgood, Ralph Phillips and Peter Sarnak [6]. In [3], Hamilton studied the following equation, which we refer to as *Hamilton's Ricci flow*

$$(*) \qquad \dot{g}(x, t) = (r - R(x, t))g(x, t), \qquad x \in M, \ t > 0,$$

where g is the metric, R is the scalar curvature of g (= twice the Gaussian curvature K), r is the average of R, and $\dot{} = \partial/\partial t$. The r in the equation above is inserted simply to preserve the area of M. He proved:

Theorem 1.1 (Hamilton). *Let (M, g) be a compact oriented Riemannian surface.*

(1) *If M is not diffeomorphic to the 2-sphere S^2, then any metric g converges to a constant curvature metric under equation $(*)$.*

(2) *If M is diffeomorphic to S^2, then any metric g with positive Gaussian curvature on S^2 converges to a metric of constant curvature under $(*)$.*

Osgood, Phillips and Sarnak have given a different proof of part (1). The object of this paper is to remove the assumption in Hamilton's theorem

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