

ON EMBEDDED COMPLETE MINIMAL SURFACES OF GENUS ZERO

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From the point of view of the classical differential geometry, embedded complete minimal surfaces of finite total curvature in \mathbf{R}^3 are interesting objects. However until the last few years no relevant results have been obtained on this field and the only known examples were the plane and the Catenoid.

Some basic properties of these surfaces were described by Jorge and Meeks in [5]. Schoen [10] characterized the Catenoid among the above surfaces as the unique one which has only two ends. The first serious effort to find nontrivial examples was made by Costa [1]. Although he was able to construct the simplest surface of this type, he gave only partial evidence of its embeddedness. The proof of this fact was obtained by Hoffman and Meeks [3] (see [2] for a complete story of this discovery) who also constructed more general examples and gave a nice characterization of those in [4].

Topologically the above surfaces are three times punctured compact surfaces of genus γ , for any $\gamma \geq 1$. No new examples of genus zero have appeared, and it is expected that such a surface does not exist. In this paper we give a proof of this fact. More precisely we will prove that:

The plane and the Catenoid are the only embedded complete minimal surfaces of finite total curvature and genus zero in \mathbf{R}^3 .

A key step in our reasoning is the proof that for any surface satisfying the hypothesis of the above result we have a one-parameter family of surfaces with the same property. This deformation is also useful in the study of the index of complete minimal surfaces of finite total curvature in \mathbf{R}^3 [8].