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KÄHLER HYPERBOLICITY AND L_2-HODGE THEORY

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0. Basic definitions and results

0.1. **Bounded and d(bounded) forms.** A differential form α on a Riemannian manifold $X = (X, g)$ is called *bounded* with respect to the Riemannian metric g if the L_∞-*norm* of α is finite,

$$\|\alpha\|_{L_\infty} \overset{\text{def}}{=} \sup_{x\in X} \|\alpha(x)\|_g < \infty.$$

We say that α is d(bounded) if α is the exterior differential of a bounded form β, i.e., $\alpha = d\beta$, where $\|\beta\|_{L_\infty} < \infty$.

Remark. It is not required that α is bounded, yet in all our applications the notion " d(bounded) " applies to bounded forms α.

If X is a compact, these notions bring nothing new. Namely, every smooth (or just continuous) form α is bounded, and α is d(bounded) if and only if it is exact. However, if X is noncompact, then an exact bounded form is not necessarily d(bounded).

0.1.A. **Example.** The form

$$\alpha = dx_1 \wedge dx_2 \wedge \cdots \wedge dx_n \text{ on } \mathbf{R}^n$$

is bounded and exact but not d(bounded).

Proof. Write $\alpha = d\beta$ and apply Stokes formula to a ball B of large radius $R \to \infty$ in $\mathbf{R}^n$. Then $\mathrm{Vol}_n B = \int_B \alpha = \int_{\partial B} \beta \leq \|\beta\|_{L_\infty} \mathrm{Vol}_{n-1} \partial B$, and $\|\beta\|_{L_\infty} \geq \mathrm{Vol}_n B / \mathrm{Vol}_{n-1} \partial B = R/n$. This shows that $\|\beta\|_{L_\infty} = \infty$; moreover, β grows at least linearly on $\mathbf{R}^n$, i.e., $\sup_{x\in\partial B} \|\beta(x)\| \geq R/n$.

0.1.B. **Hyperbolic manifolds.** (See [4], [10], [22].) If (X, g) is complete simply connected and has strictly negative sectional curvature $\sup_{x\in X} K_X(X) \leq -c < 0$, then every smooth bounded closed form α of degree $i \geq 2$ is d(bounded). This immediately follows from the well-known bound on the volume of the *geodesic cones* over the $(i-1)$-dimensional submanifolds $S \subset X$,

$$(*) \qquad \mathrm{Vol}_i(\mathrm{Cone}\, S) \leq (c(i-1))^{-1} \mathrm{Vol}_{i-1} S.$$

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