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RIGID REPRESENTATIONS OF KÄHLERIAN FUNDAMENTAL GROUPS

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0. Introduction

In the early 1960s, Weil proved that cocompact lattices in a semisimple Lie group had no deformations, provided the group had no local three-dimensional factors. This result has been the point of departure for many subsequent studies of local, strong, and super-rigidity of lattices in semisimple Lie groups. The goal in the current paper is to show that the local rigidity discovered by Weil in the context of locally symmetric manifolds is actually a phenomenon which holds much more generally in Kähler geometry.

Suppose M is a compact Kähler manifold, and $G_{\mathbf{R}}$ is a simple real algebraic group acting by isometries on the irreducible bounded symmetric domain $G_{\mathbf{R}}/K$. If P is a principal $G_{\mathbf{R}}$-bundle over M, we may reduce its structure group to K, and associate characteristic classes of the bundle with reduced structure group to P. In particular, there is an invariant connected with the volume form on $G_{\mathbf{R}}/K$, to be denoted by $\operatorname{vol}(P)$. It is a power of the first Chern class, up to a constant factor. We prove the following.

Theorem 0.1. *Suppose that P is a flat principal $G_{\mathbf{R}}$-bundle with* $\operatorname{vol}(P) \neq 0$, *and assume that $G_{\mathbf{R}}/K$ is not of the form*

$$U(n, 1)/U(n) \times U(1)$$

or

$$SO(2n+1, 2)/S(O(2n+1) \times O(2)).$$

Then the monodromy homomorphism of the fundamental group of M into $G_{\mathbf{R}}$ is locally rigid as a homomorphism into the complexification of $G_{\mathbf{R}}$.

An earlier result [4, Theorem 4.1] asserted the compactness of the space of equivalence classes of representations with $\operatorname{vol}(\rho) \neq 0$ under weaker assumptions on the bounded symmetric domain. This result supersedes

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