

ON SYMPLECTIC MANIFOLDS WITH SOME CONTACT PROPERTIES

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1. Introduction

We show in this article that a symplectic manifold bounded by the standard contact sphere is, under some additional hypotheses, a ball. This gives a tool for the recognition of nonstandard structures on spheres, and we show here that exotic contact structures on spheres of dimension > 3 do exist. These examples are of a completely different nature than Bennequin's nonstandard contact structure on S^3 (see [1]). The new nonstandard structures on high-dimensional spheres are *s-fillable* (see §3 below); i.e., they bound symplectic manifolds, while Bennequin's nonstandard structure is *overtwisted* (see [2]) and, therefore, cannot bound a symplectic manifold (see [5], [3]). In the other paper (see [3]) we show that the only *s-fillable* contact structure on S^3 is the standard one.

During the preparation of the paper the author had many stimulating discussions with D. McDuff. In particular, she proposed that the main Theorem 6.1 be deduced from the Theorem 5.1 on the structure of asymptotically flat symplectic manifolds (which is proved in her paper) instead of giving the similar proof that was originally intended. A. Floer and D. McDuff explained their proof of homological triviality of asymptotically flat symplectic manifolds. The author wants to thank them, as well as T. Januszkiewicz and C. Viterbo, for interesting discussions.

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