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ON THE MARTIN BOUNDARY OF RIEMANNIAN PRODUCTS

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Abstract

We describe the minimal Martin boundary of a Riemannian product $X = X_1 \times X_2$ where the factors are complete manifolds with Ricci curvature bounded below. As a consequence we obtain a splitting result for bounded harmonic functions.

0. Introduction

The goal of this paper is to prove a splitting theorem for positive harmonic functions on a Riemannian product, under very general assumptions: we require only that each factor be a complete, noncompact Riemannian manifold with Ricci curvature bounded below.

Given a complete, noncompact Riemannian manifold X, denote by $\lambda_0(X) \leq 0$ the supremum of the closed L^2 spectrum of the Laplace-Beltrami operator on X. For each $\lambda \geq \lambda_0$, the eigenvalue problem

$$\Delta\varphi = \lambda\varphi$$

has positive solutions. For $\lambda > \lambda_0$, $\Delta - \lambda$ is coercive, has a Green function $G^\lambda(x, y) > 0$, and the λ-eigenfunctions on an open set $U \subset X$ define a Brelot harmonic sheaf (for proofs of these facts, see [1], [20]). For each $\lambda \geq \lambda_0$, denote by $\mathcal{M}_1^\lambda(X)$ the space of minimal positive λ-eigenfunctions:

$$\mathcal{M}_1^\lambda = \{0 < f \in C^\infty(X) | \Delta f = \lambda f,\ 0 < g \leq f, \Delta g = \lambda g \Rightarrow g = (\text{const})f\}.$$

Theorem. *Let $X = X_1 \times X_2$ be a Riemannian product, where X_1 and X_2 are complete, noncompact, with Ricci curvature bounded below. Then the following hold.*

(i) *Each minimal positive harmonic function f on X splits as a product*

$$f(x) = K^{\lambda_1}(x^1)K^{\lambda_2}(x^2),$$

where $\lambda_i \geq \lambda_0(X_i)$, $K^{\lambda_i} \in \mathcal{M}_1^{\lambda_i}(X_i)$ for $i = 1, 2$, and $\lambda_1 + \lambda_2 = 0$.

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