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# POSITIVE RICCI CURVATURE ON THE CONNECTED SUMS OF $S^n \times S^m$

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## 0. Introduction and the main results

The topological implications of positive Ricci curvature turned out to be much weaker than what one has expected. For example, it has been shown in [18] that there is no upper bound on the total Betti number for complete Riemannian manifolds with $\mathrm{Ric} > 0$ in a fixed dimension, and the manifold can be of infinite topological type if it is noncompact (compare [1], [12]). In this paper, we prove some existence theorems concerning positive Ricci curvature. It also fills out a gap in [18] in dimensions 4, 5, and 6. Throughout this paper, both $n$ and $m$ will be integers $\geq 2$ and we will work in the smooth category. The main results are stated in the following theorems.

**Theorem 1.** *The connected sum* $\#_{i=1}^k S^n \times S^m$ *of k-copies of* $S^n \times S^m$ *carries a metric with* $\mathrm{Ric} > 0$ *for all* $k = 1, 2, 3, \cdots$, *where* $S^p$ *is the standard p-dimensional sphere.*

Let $M^{m+1}$ be an $(m+1)$-dimensional complete Riemannian manifold with $\mathrm{Ric} > 0$. Set

$$(1) \qquad M_k^{n,m} \equiv S^{n-1} \times \left( M^{m+1} \setminus \coprod_{i=0}^{k} D_i^{m+1} \right) \cup_{\mathrm{Id}} D^n \times \coprod_{i=0}^{k} S_i^m ,$$

where $D^n$, $D_i^{m+1}$ and $S^{n-1}$, $S_i^m$, $i = 0, 1, \cdots, k$, are balls and spheres of appropriate dimensions indicated by their superscripts, respectively. $M_k^{n,m}$ is the smooth $(n+1)$-dimensional manifold obtained by removing $(k+1)$-disjoint geodesic balls $D_i^{m+1}$, $i = 0, 1, 2, \ldots k$, in $M^{m+1}$ and then gluing $S^{n-1} \times (M^{m+1} \setminus \coprod_{i=0}^k D_i^{m+1})$ with $D^n \times \coprod_{i=0}^k S_i^m$ together by

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