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GAUSS MAPS OF SPACELIKE CONSTANT MEAN CURVATURE HYPERSURFACES OF MINKOWSKI SPACE

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Abstract

The Gauss map of a spacelike constant mean curvature hypersurface of Minkowski space is a harmonic map to hyperbolic space. The properties of such hypersurfaces are interpreted in terms of the harmonic mapping. Given an arbitrary closed set in the ideal boundary at infinity of hyperbolic space, there exists a complete entire spacelike constant mean curvature hypersurface whose Gauss map is a diffeomorphism onto the interior of the hyperbolic space convex hull of the set. Identifying ideal infinity with the light cone, this set corresponds to the lightlike directions of the hypersurface. In terms of this extrinsic data we give conditions for the hyperbolicity or parabolicity of this hypersurface. For example, if the set of lightlike directions has nonempty interior in the unit sphere, then this hypersurface can be constructed so as to admit nontrivial bounded harmonic functions. This gives many new examples of harmonic maps of the disk and the complex plane to the hyperbolic plane, which are of full rank.

There are relatively few examples of harmonic maps between noncompact manifolds. A class of such maps arises as the Gauss maps of entire spacelike constant mean curvature hypersurfaces of Minkowski space. These are harmonic maps into hyperbolic space by a verson of the Ruh-Vilms theorem. We construct certain constant mean curvature hypersurfaces and analyse their geometric and function theoretic properties. Thus, we are able to answer a question of Eells and Lemaire [17] by constructing many harmonic maps from $\mathbf{R}^2$ to the hyperbolic plane, which have rank two everywhere. We also construct maps of the hyperbolic plane into itself. A completely elementary example of family of nonconformal harmonic diffeomorphisms of the hyperbolic plane to itself has been constructed using similar methods [14]. The relation between harmonic maps and constant mean curvature surfaces has been studied by T. K. Milnor

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