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T-EQUIVARIANT K-THEORY OF GENERALIZED FLAG VARIETIES

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0. Introduction

To any (not necessarily symmetrizable) generalized $l \times l$ Cartan matrix A, one associates a Kac-Moody algebra $\mathfrak{g} = \mathfrak{g}(A)$ over $\mathbf{C}$ and group $G = G(A)$. G has a "standard unitary form" K. If A is a classical Cartan matrix, then G is a finite dimensional semi-simple simply-connected algebraic group over $\mathbf{C}$ and K is a maximal compact subgroup of G. We refer to this as the finite case. In general, one has subalgebras of $\mathfrak{g}$: $\mathfrak{h} \subset \mathfrak{b} \subseteq \mathfrak{p}$, the Cartan subalgebra, the Borel subalgebra, and a parabolic subalgebra, respectively. One also has the corresponding subgroups: $H \subset B \subseteq P$, the complex maximal torus, the Borel subgroup, and a parabolic subgroup, respectively. We denote by T the compact maximal torus $H \cap K$ of K. Let W be the Weyl group associated to $(\mathfrak{g}, \mathfrak{h})$ and let $\{r_i\}_{1 \le i \le l}$ denote the set of simple reflections. The group W operates on the compact maximal torus T (as well as on H) and hence on the group algebra $R(T) := \mathbf{Z}[X(T)]$ of the character group $X(T)$ of T and also on the quotient field $Q(T)$ of $R(T)$.

For any W-field F, we can form the smash product F_W of the group algebra $\mathbf{Z}[W]$ with F. In [19] we took, for F, the field $Q = Q(\mathfrak{h}^*)$ of all the rational functions on $\mathfrak{h}$ and defined an appropriate subring $R \subset Q_W$, and showed that R and its "appropriate" dual Λ, along with a certain R-module structure on Λ, replace the study of the cohomology algebra of G/B together with the various operators defined on $H^*(G/B)$. Hence the problem of understanding $H^*(G/B)$, especially the cup product structure and other operators on $H^*(G/B)$, reduced to a purely combinatorial (and hopefully more tractable) problem of understanding the ring R and its "dual" Λ, defined purely and explicitly in terms of the Coxeter group W and its representation on $\mathfrak{h}^*$.

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