SPACE OF SOULS IN A COMPLETE OPEN MANIFOLD OF NONNEGATIVE CURVATURE

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0. Introduction

Let M be a complete open Riemannian manifold of nonnegative curvature. The most significant result in the study of the differential structure of this type of manifold is due to Cheeger and Gromoll. In [3] they produced a totally geodesic submanifold S_0, a *soul* of M, and showed that M is diffeomorphic to the normal bundle $\nu(S_0)$ of S_0. Following this work, Sharafutdinov and, independently, Croke and Schroeder showed that there exists a *strong deformation retraction* $f: M \to S_0$ which is distance nonincreasing [4, 8]. Using this retraction one can show that if a soul is not unique, then they are all isometric and homologous to each other. Moreover, there are infinitely many isometric copies of a soul in M, which are not necessarily souls. This observation leads us to the following definition.

Definition. A subset $S \subset M$ is called a *pseudo-soul* if it is homologous and isometric to a soul S_0 with respect to the induced metric.

In particular, it is clear that all souls are pseudo-souls, and the definition is independent of a soul S_0. If a soul is not unique, then there are infinitely many pseudo-souls. The purpose of this paper is to investigate the union $\mathscr{H}$ of all pseudo-souls in M. In fact, we will prove the following theorem.

Theorem. $\mathscr{H} \subset M$ *is a totally geodesic embedded submanifold which is isometric to a product manifold* $S_0 \times N$, *where* N *is a complete manifold of nonnegative curvature diffeomorphic to a Euclidean* k-*space* $\mathbf{R}^k$ *and* k *is the dimension of the space of all parallel normal vector fields along the soul* S_0. *Furthermore any pseudo-soul in* M *is of the form* $S_0 \times \{p\}$ *for some* $p \in N$.

As an immediate corollary of this theorem, if the normal bundle itself is parallel, we obtain the splitting $M = S_0 \times N$. This special case has been independently studied in [6].

There are two trivial examples of M for which one can easily find pseudo-souls and the space $\mathscr{H}$. If M is a paraboloid, then every point