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FLOW OF NONCONVEX HYPERSURFACES INTO SPHERES

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0. Introduction

The flow of surfaces by functions of their principal curvatures has been intensively studied. It started with the work of Brakke [1], who used the formalism of geometric measure theory; a more classical approach had been chosen by Huisken, who looked at the so-called *inward flow* in [5]. The outward flow of surfaces (this term will be explained in the sequel) is scaling invariant and therefore more natural than the inward flow, producing more general results so far. Huisken [6] and Urbas [8] proved that the outward flow of *convex* surfaces by suitable functions of their principal curvatures converges to spheres. The convexity assumption is essential in their work.

In this paper we would like to present a different method for proving the convergence of star-shaped surfaces into spheres via the outward flow.

Let f be a symmetric, positive function homogeneous of degree one being defined on an open cone Γ of $\mathbf{R}^n$ with vertex in the origin, which contains the positive diagonal, i.e., all n-tuples of the form

$$(\lambda, \cdots, \lambda), \qquad \lambda \in \mathbf{R}_+. \tag{0.1}$$

Assume that

$$f \in C^0(\overline{\Gamma}) \cap C^2(\Gamma) \tag{0.2}$$

is *monotone*, i.e.,

$$\frac{\partial f}{\partial \lambda^i} > 0, \qquad i = 1, \cdots, n, \text{ in } \Gamma, \tag{0.3}$$

concave

$$\frac{\partial^2 f}{\partial \lambda^i \partial \lambda^j} \le 0, \tag{0.4}$$

and that

$$f = 0 \quad \text{on } \partial\Gamma. \tag{0.5}$$

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