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EINSTEIN METRICS

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1. Introduction

One of the natural classes of Riemannian metrics on an n-manifold is Einstein metrics. Berger and Ebin [4] were the first people to study the moduli space of Einstein metrics on an Einstein manifold. They showed that the moduli space of Einstein metrics is finite dimensional. This leads to the natural question whether the moduli space is compact. Einstein metrics have been studied by many mathematicians; there is an excellent book on Einstein manifolds [6].

One of the main accomplishments of this paper is that we found a compactness property of moduli spaces of four-dimensional Einstein manifolds. To explain this, let us start with a 4-manifold M, which has an Einstein metric. We consider the moduli space of all Einstein metrics on M, and normalize the Einstein metrics such that the Ricci curvature equal $+3$, -3, or 0. Let $G(M)$ be the subspace of all normalized Einstein metrics on M with the injective radius bounded from below by a fixed constant $i_0 > 0$ and diameter bounded from above by d. We are able to show that $G(M)$ is compact as a subset of the moduli space of Einstein metrics in C^∞-topology.

Theorem 1.1. *The subset of normalized Einstein metrics with Ricci curvature equal to three and with injectivity radius bounded from below on a 4-manifold M is compact in C^∞-topology.*

Theorem 1.2. *The subset of normalized Einstein metrics with Ricci curvature equal to negative three or zero, with injectivity radius bounded from below and diameter bounded from above on a 4-manifold M is compact in C^∞-topology.*

Remark 1.3. It seems that without the lower bound of the injectivity radius, the results are false.

We briefly describe here the method used in this paper. For a sequence of Einstein manifolds (M_k) with proper restrictions, by passing to a sub-

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