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# ON A SET OF POLARIZED KÄHLER METRICS ON ALGEBRAIC MANIFOLDS

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## 0. Introduction and statement of main theorems

A projective algebraic manifold $M$ is a complex manifold in certain projective space $CP^N$, $N \geq \dim_C M = n$. The hyperplane line bundle of $CP^N$ restricts to an ample line bundle $L$ on $M$. This bundle $L$ is a polarization on $M$. For the Kähler metric $g$ on $M$, we can associate a positive, $d$-closed $(1, 1)$-form $\omega_g$. In any local coordinate system $(z_1, \cdots, z_n)$ of $M$, the metric $g$ is expressed by a tensor $(g_{i\bar{j}})_{1\leq i, j\leq n}$, and $\omega_g$ is defined to be $\frac{\sqrt{-1}}{2\pi}\sum_{i,j=1}^n g_{i\bar{j}} dz_i \wedge d\bar{z}_j$. We call this $\omega_g$ the Kähler form associated to the metric $g$. By a polarized Kähler metric with respect to $L$, we mean a Kähler metric with its associated Kähler form representing the Chern class $C_1(L)$ of $L$ in $H^2(M, Z)$. We denote by $\mathrm{Ka}(M)$ the set of all polarized Kähler metrics on $M$ with respect to $L$. Given a Kähler metric $g$ in $\mathrm{Ka}(M)$, one can find a hermitian metric $h$ on $L$ with its Ricci curvature form equal to $\omega_g$ (cf. [7], or Lemma 1.1 in §1). For each positive integer $m > 0$, the hermitian metric $h$ induces a hermitian metric $h^m$ on $L^m$. Choose an orthonormal basis $\{S_0^m, \cdots, S_{N_m}^n\}$ of the space $H^0(M, L^m)$ of all holomorphic global sections of $L^m$. Here the inner product on $H^0(M, L^m)$ is the natural one induced by the Kähler metric $g$ and the hermitian metric $h^m$ on $L^m$, i.e., $\langle S_\alpha^m, S_\beta^m\rangle = \int_m h^m(S_\alpha^m, S_\beta^m)\, dV_g$. Such a basis $S_0^m(x), \cdots, S_{N_m}^n(x)$ induces a holomorphic embedding $\varphi_m$ of $M$ into $CP^{Nm}$ by assigning the point $x$ of $M$ to $[S_0^m(x), \cdots, S_{N_m}^m(x)]$ in $CP^{N_m}$. Let $g_{\mathrm{FS}}$ be the standard Fubini-Study metric on $CP^{N_m}$, i.e., $\omega_{g_{\mathrm{FS}}} = \frac{\sqrt{-1}}{2\pi}\partial\bar{\partial}\log(\sum_{i=0}^{N_m}|w_i|^2)$ for a homogeneous coordinate system $[w_0, \cdots, w_{N_m}]$ of $CP^{N_m}$. The $\frac{1}{m}$-multiple of $g_{\mathrm{FS}}$ on $CP^{N_m}$ restricts to a Kähler metric $\frac{1}{m}\varphi_m^* g_{\mathrm{FS}}$ on $M$. This metric is obviously in $\mathrm{Ka}(M)$, i.e., polarized by $L$, and it is called

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