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GAUSSIAN MAPS ON ALGEBRAIC CURVES

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0. Introduction

Let C be a complete nonsingular curve over $\mathbf{C}$ and let L be a line bundle of positive degree. We have previously considered [8] the natural map

$$\Phi_L\colon \Lambda^2 H^0(L) \to H^0(\Omega^1_C \otimes L^2),$$

defined essentially by

$$f \wedge g \mapsto f\,dg - g\,df.$$

If $C \subset \mathbf{P}^n$ is an embedding and $L = \mathscr{O}_C(1)$, one may consider the Gauss mapping

$$C \to \operatorname{Grass}(1, n),$$

associating to each point its tangent line in $\mathbf{P}^n$. Composing with the Plucker embedding of the Grassmannian into $\mathbf{P}^N$ gives the "associated curve" $\psi\colon C \to \mathbf{P}^N$. One checks that restriction of the hyperplane section $\psi^*\colon H^0(\mathbf{P}^N, \mathscr{O}_{\mathbf{P}^N}(1)) \to H^0(C, \psi^*\mathscr{O}(1))$ for this map gives Φ_L (note $H^0(\mathbf{P}^N, \mathscr{O}_{\mathbf{P}}(1)) \simeq \wedge^2 H^0(\mathbf{P}^n, \mathscr{O}(1)) \simeq \wedge^2 H^0(C, L))$. For this reason we call Φ_L or its generalization a *Gaussian* map, and its image the Gaussian linear series.

The original interest in these maps arose from studying Φ_K, where K is the canonical bundle on a smooth curve (Φ_K has been named the "Wahl map" by certain authors [3]).

Theorem 1 [8]. *If the smooth curve C lies on a K-3 surface, then Φ_K is not surjective.*

Theorem 2 [8]. *If $C \subset \mathbf{P}^n$ is a complete intersection, with multidegrees $d_1 \le d_2 \le \cdots \le d_{n-1}$ $(d_1 \ge 2)$, then Φ_K is surjective if $d_1 + \cdots + d_{n-2} > n+1$.*

Theorem 3 [3]. *For a general curve C of genus 10 or ≥ 12, Φ_K is surjective.*

Theorem 1 gives the only known intrinsic property which a curve must satisfy in order to sit on a K-3 surface. Our original proof involved the

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