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A RIGIDITY THEOREM FOR PROPERLY EMBEDDED MINIMAL SURFACES IN $\mathbf{R}^3$

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Abstract

We consider the question of when an intrinsic isometry of a properly embedded minimal surface is induced by an ambient isometry. We prove it always extends when the surface has at least two ends.

There are several interesting theorems and conjectures on the rigidity of complete surfaces in $\mathbf{R}^3$, which satisfy some natural geometric constraint. Perhaps the best known theorem of this type is the Cohn-Vossen theorem which shows that a compact Riemannian surface of positive curvature has a unique immersion into $\mathbf{R}^3$. A similar result is conjectured for complete surfaces of nonpositive or nonnegative curvature with the additional hypothesis that the surface has finite area. It is also conjectured that tight surfaces are rigid.

In this paper we prove a rigidity theorem for properly embedded minimal surfaces in $\mathbf{R}^3$, which have more than one end. This theorem states that the inclusion map of the surface into $\mathbf{R}^3$ represents the unique isometric minimal immersion of such a surface up to a rigid motion of $\mathbf{R}^3$. In particular it follows from our theorem that every intrinsic isometry of this type of surface extends to an isometry of $\mathbf{R}^3$.

In §1 we derive a geometric condition on an immersed minimal surface which guarantees that the surface is minimally rigid. We prove that if the surface intersects a plane transversally along an immersed closed curve, then any other isometric minimal immersion of the surface into $\mathbf{R}^3$ differs from the original immersion by an ambient isometry. In §2 we obtain some asymptotic properties of solutions of the minimal surface equation over annular planar domains. In §3 we prove that if a properly embedded minimal surface M in $\mathbf{R}^3$ has more than one end, then M is transverse

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