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A STRUCTURE THEOREM FOR HOLOMORPHIC CURVES IN $\mathrm{Gr}(3, \mathbf{C}^6)$

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Abstract

A holomorphic curve f in $\mathrm{Gr}(n, \mathbf{C}^{2n})$ is called generic if the curvature of the canonical connection of $f^*(S(n, \mathbf{C}^{2n}))$ has distinct eigenvalues, where $S(n, \mathbf{C}^{2n})$ is the universal subbundle over $\mathrm{Gr}(n, \mathbf{C}^{2n})$. A holomorphic curve in $\mathrm{Gr}(n, \mathbf{C}^{2n})$ is completely split if it is the orthogonal direct sum of n holomorphic curves in the projective plane. These two types of curves are both relatively simple. In this paper, we prove that a 1-nondegenerated holomorphic curve in $\mathrm{Gr}(3, \mathbf{C}^6)$ is either generic or completely split.

Introduction

Denote the Grassmannian of n-dimensional subspaces of $\mathbf{C}^{2n}$ by $\mathrm{Gr}(n, \mathbf{C}^{2n})$. A holomorphic curve in $\mathrm{Gr}(n, \mathbf{C}^{2n})$ is locally a holomorphic mapping of some open disk in $\mathbf{C}$ into $\mathrm{Gr}(n, \mathbf{C}^{2n})$. Because of the analytic structure, we can restrict ourselves to the local holomorphic curves only.

Let $f: \Omega \to \mathrm{Gr}(n, \mathbf{C}^{2n})$ be a holomorphic curve. For each z in Ω, we define $(f(z), f'(z)) = \mathrm{span}\{\gamma_1(z), \cdots, \gamma_n(z), \gamma_1'(z), \cdots, \gamma_n'(z)\}$, where $\gamma_j: \Omega \to \mathbf{C}^{2n}$ is holomorphic and $\mathrm{span}\{\gamma_1(z), \cdots, \gamma_n(z)\} = f(z)$. Clearly, (f, f') is independent of the choice of $\gamma_1, \cdots, \gamma_n$. We say f is 1-nondegenerated if $(f(z), f'(z)) = \mathbf{C}^{2n}$ for each $z \in \Omega$.

Throughout this paper, by "holomorphic curve" we mean "1-nondegenerated holomorphic curve". Let f be a 1-nondegenerated holomorphic curve in $\mathrm{Gr}(n, \mathbf{C}^{2n})$. Then the holomorphic Hermitian vector bundle

$$E_f: \begin{array}{c} \text{the space } f(z) \\ \downarrow \\ z \end{array}$$

is a completely unitary invariant of f by the Calabi rigidity theorem. By

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