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# CRITICAL POINTS OF YANG-MILLS FOR NONCOMMUTATIVE TWO-TORI

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In [5] A. Connes and the author described the moduli spaces for the minima of the Yang-Mills function for the case of connections on projective modules over noncommutative two-tori, in the setting of the noncommutative differential geometry initiated by Connes in [4]. The main purpose of the present note is to describe the critical points of the Yang-Mills function for the same case, and also the moduli spaces for these critical points. It turns out that the critical points coincide with certain connections which were used in [3] to construct actions of the Heisenberg Lie group on noncommutative tori. (In fact, we will make crucial use of one of the arguments from [3].) We will find that the moduli spaces for the critical points are finite products of the kinds of spaces which were obtained in [5] as moduli spaces for the minima.

## 1. The Yang-Mills equations

We begin by recalling briefly the setting of [5]. Let $G$ be a Lie group, and let $\alpha$ be an action of $G$ as automorphisms of a $C^*$-algebra $A$. We let $A^\infty$ be the dense $*$-subalgebra of $A$ consisting of the $C^\infty$-vectors for $\alpha$. Then the infinitesimal form of $\alpha$ gives an action, $\delta$, of the Lie algebra, $L$, of $G$, as derivations of $A^\infty$. Every finitely generated projective right $A$-module $\Xi$ has a $C^\infty$-version $\Xi^\infty$. Since we will never work with $A$ or $\Xi$, but only with $A^\infty$ and $\Xi^\infty$, we will for notational simplicity denote the latter by $A$ and $\Xi$ from now on. Also, for brevity we will say "projective" when we mean "finitely generated projective".

We can and will assume that $\Xi$ is equipped with a Hermitian metric, $\langle\, ,\, \rangle_A$, that is, an $A$-valued inner product for which it is self-dual. The effect of the choice of Hermitian metric on what follows is discussed in [5, p. 241]. In the role of Riemannian metric for $A$ we assume that $L$ is equipped

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