J. DIFFERENTIAL GEOMETRY
31 (1990) 473-499

GEOMETRIC PROPERTIES OF MAPPINGS BETWEEN HYPERSURFACES IN COMPLEX SPACE

M. S. BAOUENDI & LINDA PREISS ROTHSCHILD

Table of Contents

0. Introduction

We consider formal, smooth, or real analytic hypersurfaces in $\mathbb{C}^{n+1}$ and mappings between such hypersurfaces. The mappings we study are either the restrictions of germs of holomorphic mappings, CR mappings, or, more generally, formal holomorphic mappings defined by nonconvergent power series. Let M be given locally by $\rho(Z,\overline{Z}) = 0$, with $Z \in \mathbb{C}^{n+1}$, ρ real, $\rho(0) = 0$, $d\rho(0) \neq 0$. A *transversal* coordinate for M is Z_{n+1} defined by $\rho(Z,0) = \alpha(Z)Z_{n+1}$, $\alpha(0) \neq 0$. If $H\colon M \to M'$ is a holomorphic or formal map between two hypersurfaces M, M' in $\mathbb{C}^{n+1}$, given by $Z_i' = H_i(Z)$, and if Z_{n+1}' is a transversal coordinate for M', then H_{n+1} is called a *transversal component* of H.

Our first result in this paper (Theorem 1) shows that if M' is of finite type (in the sense of Kohn [12] and Bloom-Graham [5]) and H is of finite multiplicity (as in [3]), then $(\partial H_{n+1}/\partial Z_{n+1})(0) \neq 0$, where H_{n+1} and Z_{n+1} are transversal. This was proved by Fornaess [9] in the pseudoconvex case using the Hopf Lemma. We also show (Theorem 2) that if M is essentially finite (as defined in [2], [3] and [6]) and H_{n+1} does not vanish identically, then H is of finite multiplicity. Next we show (Theorem 3) that if M and M' are essentially finite, then H is of finite multiplicity if and only if a certain Jacobian determinant associated to H is nonvanishing.

Received November 16, 1988. The first author was supported by National Science Foundation Grant DMS-8603176, and the second author DMS-8601260.