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MODIFIED DEFECT RELATIONS FOR THE GAUSS MAP OF MINIMAL SURFACES. II

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1. Introduction

Let $x = (x_1, \cdots, x_m)\colon M \to \mathbf{R}^m$ be a (connected, oriented) minimal surface immersed in a Euclidean m-space $\mathbf{R}^m$ ($m \geq 3$). We denote the set of all oriented 2-planes in $\mathbf{R}^m$ by Π. For each $P \in \Pi$ taking a positive orthonormal basis (X, Y) of P and setting $Z := (X - iY)/2$ in a complex number m-space $\mathbf{C}^m$, we assign the point $\Phi(P) := \pi(Z)$, where π denotes the canonical projection of $\mathbf{C}^m - \{0\}$ onto the complex projective space $P^{m-1}(\mathbf{C})$. Then the map $\Phi\colon \Pi \to P^{m-1}(\mathbf{C})$ maps Π bijectively onto the quadric

$$Q_{m-2}(\mathbf{C}) := \{(w_1 : \cdots : w_m); w_1^2 + \cdots + w_m^2 = 0\}.$$

For a point $p \in M$ the tangent plane $T_p(M)$ of M at p is considered an oriented 2-plane in $\mathbf{R}^m$, where $T_p(\mathbf{R}^m)$ is identified with $\mathbf{R}^m$ by the parallel translation which maps p to the origin. By definition, the (generalized) Gauss map of M is the map $G\colon M \to Q_{m-2}(\mathbf{C})$ ($\subset P^n(\mathbf{C})$) which maps each point $p \in M$ to the point $\Phi(T_p(M))$, where $n = m - 1$. The metric induced from $\mathbf{R}^m$ gives a conformal structure on M, and M is considered a Riemann surface. By the assumption of minimality of M, G is a holomorphic map of M into $P^n(\mathbf{C})$. In the case $m = 3$, $Q_1(\mathbf{C})$ can be identified with the Riemann sphere, and G is considered a meromorphic function, whose conjugate is the classical Gauss map of M.

In 1981, F. Xavier showed that the Gauss map of a nonflat complete minimal surface in $\mathbf{R}^3$ could not omit 7 points of the sphere [13]. Afterwards, as a generalization of this, the author proved that, if the Gauss map G of a complete minimal surface M in $\mathbf{R}^m$ is nondegenerate, namely, $G(M)$ is not contained in any hyperplane in $P^{m-1}(\mathbf{C})$, then it can omit at most m^2 hyperplanes in general position [4]. Moreover, in [5] and [6] he gave several improvements of this result. Recently, the author has improved F. Xavier's result by showing that the Gauss map of a nonflat complete

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