J. DIFFERENTIAL GEOMETRY
31 (1990) 343-355

ON THE GAUSS MAP AND TOTAL CURVATURE OF COMPLETE MINIMAL SURFACES AND AN EXTENSION OF FUJIMOTO'S THEOREM

XIAOKANG MO & ROBERT OSSERMAN

1. Introduction

Our object in this paper is to put into final form the connection between total curvature and the Gauss map of complete minimal surfaces in $\mathbf{R}^3$ and $\mathbf{R}^4$. The initial results were obtained by Osserman [11], [12] in 1963. We combine the methods used there with recent ideas of Fujimoto [5] and intermediate results of Hoffman and Osserman [7].

The inspiration for this paper was Fujimoto's Theorem [5]: *Let S be a complete minimal surface in $\mathbf{R}^3$, not a plane. Then the image of S under the Gauss map can omit at most four points on the unit sphere.*

There are a number of examples showing that the number "four" in this theorem is best possible (see [14, pp. 72–74]). An earlier paper by Xavier [15] was important in that it was the first to show that at most a finite number of points could be omitted. Using Xavier's approach, Earp and Rosenberg [3] were able to obtain results in the direction of those given here, for the case of surfaces in $\mathbf{R}^3$, under the assumption of finite topological type. However, Xavier's method does not seem able to yield the optimal results.

Fujimoto also provided an analog of his theorem for minimal surfaces in $\mathbf{R}^4$.

Among the results obtained in this paper are the following.

Theorem 1. *Let S be a complete minimal surface in $\mathbf{R}^3$. If the Gauss map g takes on five distinct values only a finite number of times, then S has finite total curvature.*

The behavior of the Gauss map for a complete minimal surface of finite total curvature is known in great detail [12]. In particular, the image under the Gauss map cannot omit more than three values. Thus, one consequence of the above theorem is a sharpening of Fujimoto's Theorem:

Received March 28, 1988. This research was supported in part by National Science Foundation grant No. DMS84-08725.