J. DIFFERENTIAL GEOMETRY
31 (1990) 285-299

ASYMPTOTIC BEHAVIOR FOR SINGULARITIES OF THE MEAN CURVATURE FLOW

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Let M^n, $n \geq 1$, be a compact n-dimensional manifold without boundary and assume that $\mathbf{F}_0: M^n \to \mathbb{R}^{n+1}$ smoothly immerses M^n as a hypersurface in a Euclidean $(n+1)$-space $\mathbb{R}^{n+1}$. We say that $M_0 = \mathbf{F}_0(M^n)$ is moved along its mean curvature vector if there is a whole family $\mathbf{F}(\bullet, t)$ of smooth immersions with corresponding hypersurfaces $M_t = \mathbf{F}(\bullet, t)(M^n)$ such that

$$\begin{aligned} \frac{\partial}{\partial t}\mathbf{F}(p,t) &= \mathbf{H}(p,t), \qquad p \in M^n, \\ \mathbf{F}(\cdot, 0) &= \mathbf{F}_0 \end{aligned} \tag{1}$$

is satisfied. Here $\mathbf{H}(p,t)$ is the mean curvature vector of the hypersurface M_t at $\mathbf{F}(p,t)$. We saw in [7] that (1) is a quasilinear parabolic system with a smooth solution at least on some short time interval. Moreover, it was shown that for convex initial data M_0 the surfaces M_t contract smoothly to a single point in finite time and become spherical at the end of the contraction.

Here we want to study the singularities of (1) which can occur for nonconvex initial data. Our aim is to characterize the asymptotic behavior of M_t near a singularity using rescaling techniques. These methods have been used in the theory of minimal surfaces and more recently in the study of semilinear heat equations [3], [4]. An important tool of this approach is a monotonicity formula, which we establish in §3. Assuming then a natural upper bound for the growth rate of the curvature we show that after appropriate rescaling near the singularity the surfaces M_t approach a selfsimilar solution of (1). In §4 we consider surfaces M_t, $n \geq 2$, of positive mean curvature and show that in this case the only compact selfsimilar solutions of (1) are spheres. Finally, in §5 we study the model-problem of a rotationally symmetric shrinking neck. We prove that the natural growth rate estimate is valid in this case and that the rescaled solution asymptotically approaches a cylinder.

Received April 1, 1988.