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THE TOPOLOGICAL UNIQUENESS OF TRIPLY PERIODIC MINIMAL SURFACES IN R^3

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0. Introduction

The study of topological uniqueness questions related to minimal surfaces was initiated by Lawson in [7]. Lawson proved that if F_1 and F_2 are embedded minimal surfaces in the three-sphere having the same genus, then there is a homeomorphism of the three-sphere taking F_1 to F_2. The method of proof is to show that an embedded minimal surface in a three-manifold of positive Ricci curvature is a Heegaard splitting, and then appeal to a theorem of Waldhausen stating that any two Heegaard splittings of the three-sphere having the same genus are topologically equivalent.

The study of the topological uniqueness of minimal surfaces was furthered by Meeks; a good source for this material is his IMPA lecture notes [8]. Meeks proved that if F is an embedded minimal surface in a closed flat three-manifold, then F is either totally geodesic or a Heegaard splitting. Using this fact he proved that if F_1 and F_2 are minimal surfaces having one boundary component and the same genus in a flat three-ball with convex boundary, then the surfaces are topologically equivalent. Once again the proof is by appeal to Waldhausen's theorem.

In [5] the author showed that any two genus-three minimal surfaces in a flat three-torus are topologically equivalent. The proof is a topological analysis of genus-three Heegaard splittings of T^3 that can be minimal surfaces. In [6] it is shown that genus-three Heegaard splittings of the three-torus are topologically unique. The method of proof is to obtain a minimax surface from the isotopy class of the Heegaard splitting using a technique of Pitts and Rubinstein [9]. The topological uniqueness of genus-three Heegaard splittings can be determined by analyzing the resulting minimal surface.

At the end of [8], Meeks gives a list of fifty open problems in minimal surface theory. Many of them pertain to the topology of minimal surfaces.

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