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# A PROOF OF THE SPLITTING CONJECTURE OF S.-T. YAU

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**Abstract**

According to a conjecture of Yau, a geodesically complete space-time satisfying the timelike convergence condition and admitting a timelike line is isometric to the product of that line and a spacelike hypersurface. A proof for globally hyperbolic space-times has recently been provided by Eschenberg. The present paper shows how Eschenburg's arguments may be refined so as to yield a complete proof of Yau's conjecture.

## 1. Introduction

The Cheeger-Gromoll [4] splitting theorem of Riemannian geometry states that a complete Riemannian $(n + 1)$-manifold of nonnegative Ricci curvature admitting an absolutely maximizing geodesic is isometric to the product of that geodesic and a complete Riemannian $n$-manifold. The simplest known proof of this result is due to Eschenburg and Heintze [6]. The following conjecture of Yau proposed that there should be an analogue of the Cheeger-Gromoll theorem for Lorentzian manifolds.

**Conjecture**(*Yau* [10]). *A geodesically complete Lorentzian* 4*-manifold of nonnegative Ricci curvature in the timelike direction which contains an absolutely maximizing timelike geodesic is isometrically the cross product of that geodesic and a spacelike hypersurface.*

Various restricted results pertaining to this conjecture were first established by Galloway [7] and by Beem et al. [2], [3]. However the central development came with a recent paper of Eschenberg [5]. This showed that the elementary proof of the Cheeger-Gromoll theorem given in [6] could be directly translated to the Lorentzian case to give a proof of the Yau conjecture, subject to an additional hypothesis of global hyperbolicity. A subsequent paper of Galloway [8] employed maximal surface techniques to obtain a more natural and more powerful approach to the key step in Eschenberg's theorem. This led to a demonstration that, if global hyperbolicity does hold, then the hypothesis of timelike geodesic completeness

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