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NUMERICAL ALGORITHMS FOR PROPAGATING INTERFACES: HAMILTON-JACOBI EQUATIONS AND CONSERVATION LAWS

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Abstract

In many physical problems, interfaces move with a speed that depends on the local curvature. Some common examples are flame propagation, crystal growth, and oil-water boundaries. We idealize the front as a closed, nonintersecting, initial hypersurface flowing along its gradient field with a speed that depends on the curvature. Because explicit solutions seldom exist, numerical approximations are often used. In this paper, we review some recent work on algorithms for attacking these problems. We show that algorithms based on direct parametrizations of the moving front face considerable difficulties. This is because such algorithms adhere to local properties of the solution, rather than the global structure. Conversely, the global properties of the motion can be captured by embedding the surface in a higher-dimensional function. In this setting, the equations of motion can be solved using numerical techniques borrowed from hyperbolic conservation laws. We apply the algorithms to a variety of complicated shapes, showing corner formation and breaking and merging, and conclude with a study of a dumbbell in R^3 moving under its mean curvature. We follow the collapsing dumbbell as the handle pinches off, a singularity develops, and the front breaks into two separate surfaces.

In many physical problems, interfaces move with speed that depends on the local curvature. Explicit solutions seldom exist. Thus, there is great interest in numerical algorithms that approximate the position of the moving front. In this paper, we review some recent work on algorithms for attacking these problems. The goal of this paper is to show that algorithms based on direct parametrizations of the moving front face considerable difficulties. This is because such algorithms adhere to local properties of the solution, rather than the global structure. Conversely, the

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