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THE BIHOLOMORPHIC CURVATURE OF QUASISYMMETRIC SIEGEL DOMAINS

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0. Introduction

Let D be a bounded homogeneous domain in $\mathbb{C}^N$ equipped with the canonical Kähler-Einstein Bergman metric. Then D has nonpositive sectional curvature if and only if D is symmetric [6]. For complex manifolds natural generalizations of the sectional curvature are the holomorphic sectional curvature and the holomorphic bisectional curvature as defined in [10]. In this paper we want to investigate the holomorphic bisectional curvature for quasisymmetric Siegel domains, a class of homogeneous Siegel domains which lies strictly between the classes of symmetric domains and general bounded homogeneous domains. Perhaps the simplest characterization of the irreducible quasisymmetric domains is that their Bergman metric induces a symmetric metric on the tube subdomain [4]. A formula for the holomorphic bisectional curvature for quasisymmetric Siegel domains has been given in [15]—using the classification of quasisymmetric Siegel domains and a case-by-case argument. It was shown in [15], however, only that the holomorphic sectional curvature is nonpositive for quasisymmetric Siegel domains.

In the present paper we give a classification free proof of Zelow's formula for the holomorphic bisectional curvature and—as its main result—show that the holomorphic bisectional curvature of quasisymmetric Siegel domains is always nonpositive (§4.14). We would like to note that this is in contrast with a recent paper of Mok-Zhong [11] which states that a *compact* Kähler-Einstein manifold of nonnegative holomorphic bisectional curvature and positive Ricci curvature is isometric to a Hermitian symmetric space. Thus there is no direct analog of the Mok-Zhong result in the noncompact case.

The plan of this paper is as follows. One observes (in §4) that all irreducible quasisymmetric Siegel domains of rank greater than 2 occur as Kähler submanifolds of symmetric Siegel domains (in fact, as fibers of a

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