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POSITIVE SCALAR CURVATURE AND LOCAL ACTIONS OF NONABELIAN LIE GROUPS

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1. Introduction

Lawson and Yau proved in [7] that if a compact, connected, nonabelian Lie group G acts smoothly and effectively on a compact manifold M, then M admits a riemannian metric of positive scalar curvature. In Theorem A below we show that the same conclusion holds under somewhat weaker assumptions described by the following definition:

1.1. Definition. A local action of nonabelian Lie groups (or $\mathscr{N}$-structure) on a smooth manifold M consists of a finite cover $(U_i)_{i\in I}$ of M by open, connected sets U_i and a family $F_i\colon G_i \times U_i \to U_i$ $(i \in I)$ of smooth, effective actions of compact, connected, nonabelian Lie groups G_i such that the following compatibility condition holds:

> for $i, j \in I$ the set $U_{ij} = U_i \cap U_j$ (if nonempty) is both G_i- and G_j-invariant and one of the two groups contains the other if we treat them as subgroups of $\operatorname{Homeo}(U_{ij})$.

Theorem A. *If a compact manifold M admits a local action by nonabelian Lie groups, then it admits a riemannian metric of positive scalar curvature.*

§§4 and 5 contain the main conceptual body of the proof of Theorem A and explain its relation to [7]. The technical core of the proof is deferred to §§9 and 10.

Theorem B (see §2) states that *if M and N are two manifolds with $\mathscr{N}$-structures and* $\dim(M) = \dim(N) \geq 6$, *then the connected sum $M\#N$ also has an $\mathscr{N}$-structure.* This theorem thus provides a method of constructing local actions from global ones and illustrates some flexibility of $\mathscr{N}$-structures, which is not shared by global actions.

Theorem C (see §3) supplies examples of manifolds (with the family $(T^n \times S^2)\#(T^n \times S^2)$, $n \geq 3$, among them) which admit local actions but no global action by a nonabelian group. As those manifolds have metrics

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