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COMPACTIFYING COVERINGS OF CLOSED 3-MANIFOLDS

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Let M be a closed P^2-irreducible 3-manifold with infinite fundamental group. It is a longstanding conjecture that the universal cover of M must be homeomorphic to $\mathbf{R}^3$. Waldhausen [17] proved that this is the case where M is Haken. (See Heil's paper [5] for the nonorientable case.) The first result of this paper is the following generalization of Waldhausen's result.

Theorem 1.1. *Let M be a closed P^2-irreducible 3-manifold. If $\pi_1(M)$ contains the fundamental group of a closed surface other than S^2 or P^2, then the universal cover of M is homeomorphic to $\mathbf{R}^3$.*

We will say that a 3-manifold is *almost compact* if it can be obtained from a compact manifold N by removing a closed subset of ∂N. Then Theorem 1.1 is equivalent to the assertion that the universal covering of M is almost compact. A natural way in which to attempt to generalize Theorem 1.1 is to show that other coverings of M are almost compact. It was conjectured by Simon [14] that if M is any compact P^2-irreducible 3-manifold, and M_1 is a covering of M with finitely generated fundamental group, then M_1 must be almost compact. Simon verified this conjecture for the case where $\pi_1(M_1)$ is the fundamental group of a boundary component of M. Jaco [6] generalized this to the case where $\pi_1(M_1)$ is a finitely generated peripheral subgroup of $\pi_1(M)$. More recently, Thurston [15] showed that if M admits a geometrically finite complete hyperbolic structure of infinite volume, then Simon's conjecture is true. Finally, Bonahon [1] showed that any hyperbolic 3-manifold M with finitely generated fundamental group is almost compact provided $\pi_1(M)$ is not a free product.

The second result of this paper is the following.

Theorem 2.1. *Let M be a closed P^2-irreducible 3-manifold such that $\pi_1(M)$ contains a subgroup A isomorphic to $\mathbf{Z}\times\mathbf{Z}$. Then the covering of M with fundamental group A is almost compact.*

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